6.1 Quadrilaterals: Parallelograms, Rectangles, Rhombi, Squares

1. Write the definition of a parallelogram as a biconditional statement using the phrase “…if and only if…” What other special quadrilaterals can be classified as parallelograms?

2. List five properties that are true of a parallelogram’s sides, angles, and diagonals. (I.e., the opposite sides are congruent.)

3. The measure of one interior angle of a parallelogram is 0.25 times the measure of another angle. Find the measure of each angle.

4. The sides of parallelogram \(WXYZ\) are represented by the expressions listed below. Sketch parallelogram \(WXYZ\), and find its perimeter.
\[
WZ = -2x + 37 \quad WX = 4y + 5 \quad XY = x - 5 \quad ZY = y + 14
\]

5. Find the missing lengths and angle measurements of parallelogram \(ABCD\) given the measurements on the diagram and diagonal lengths \(AC = 7.8\) and \(EB = 5.4\).

6. Points \(J(1, 2)\), \(K(3, 6)\), and \(L(6, 4)\) are three vertices of a parallelogram. Find the possible coordinates that could be the fourth point \(M\).

7. Definitions:
   a. Write the definitions of a rectangle, a rhombus, and a square.
   b. Based on these definitions, does a square classify as a rhombus and/or a rectangle? Explain.
   c. Draw a Venn diagram that shows the relationships between a parallelogram, rectangle, rhombus, and square.
8. What is the relationship between the diagonals in rectangles and squares? What kinds of triangles are then formed in the interior of these quadrilaterals?

9. Find all of the missing lengths and angle measures for rectangle $FGHJ$ given the measurements on the diagram and diagonal length $FH = 7$. Here are the parts to find:

$$\begin{align*}
FG &= 6.3 \\
\overline{GH} &= 3 \\
\text{JG} &= \\
\text{JK} &= \\
\text{FJ} &= \\
\text{JH} &= \\
\text{KH} &= \\
\text{m}\angle \text{KJH} &= \\
\text{m}\angle \text{KHJ} &= \\
\text{m}\angle \text{JKH} &= \\
\text{m}\angle \text{HKG} &= \\
\text{m}\angle \text{KGH} &= \\
\text{m}\angle \text{GHK} &= \\
\text{m}\angle \text{GFK} &= \\
\text{m}\angle \text{FKG} &= \\
\text{m}\angle \text{KJF} &= \\
\text{m}\angle \text{JKF} &= \\
\text{m}\angle \text{GZU} &= \\
\text{m}\angle \text{UQZ} &= \\
\text{m}\angle \text{QUZ} &= \\
\text{m}\angle \text{UZD} &= \\
\text{m}\angle \text{ZDA} &= \\
\text{m}\angle \text{QZD} &= \\
\text{m}\angle \text{DUA} &= \\
\text{m}\angle \text{QAU} &= \\
\end{align*}$$

10. What type of quadrilateral is $ABCD$ on the coordinate grid shown at right. Confirm your answer with numerical evidence.

11. For square $QUAD$, find all of the missing lengths and angles measurements.

$$\begin{align*}
\text{QA} &= \\
\text{QD} &= \\
\text{DA} &= \\
\text{AZ} &= \\
\text{DU} &= \\
\text{m}\angle \text{QZU} &= \\
\text{m}\angle \text{UQZ} &= \\
\text{m}\angle \text{QUZ} &= \\
\text{m}\angle \text{QZD} &= \\
\text{m}\angle \text{ZDQ} &= \\
\text{m}\angle \text{QAD} &= \\
\text{m}\angle \text{UZA} &= \\
\text{m}\angle \text{ZDA} &= \\
\text{m}\angle \text{DUA} &= \\
\text{m}\angle \text{ZQD} &= \\
\text{m}\angle \text{QAU} &= \\
\end{align*}$$
6.2 Quadrilaterals: Kites, Trapezoids, Midsegments

1. Define and list the properties of a kite and trapezoid.

2. What properties are true of isosceles trapezoids that are not true of trapezoids that are not isosceles?

3. Construct kite $ABCD$ such that $AB \cong BC$ and $CD \cong AD$.

4. Explain how could you draw a kite $EFGH$ by first creating the diagonals?

5. Suppose there is an isosceles trapezoid named $IJKL$ with $IJ \parallel JK$. If $IK = 9x + 7$ units, and $JL = 13x - 9$ units, what is the value of $x$?

6. In each quadrilateral, find the measure of $\angle G$. If possible, draw in a line of symmetry for each shape.

7. Construct a triangle and reflect it to create a kite. What other quadrilaterals does this technique work with? Does a kite have rotational symmetry?
8. For each figure, find the values of $x$ and $y$.
   a. The perimeter is 160 cm.
   b. 

   

9. $\text{MNOP}$ is an isosceles trapezoid with $\overline{NO} \cong \overline{MP}$. If $m\angle M = (4x)\degree$ and $m\angle P = (x + 21)\degree$, find the value of $x$.

10. Draw a trapezoid, and create the midsegment connecting its legs. Next, draw a triangle and one of its medians. How is the midsegment like the median? How is it different? Use your diagrams to explain your reasoning.

11. Find the missing variable.
   a. $h = \boxed{13}$
   b. $k = \boxed{5.86}$
   c. $x = \boxed{21}$
   d. outside perimeter = \boxed{8.12}
6.3 Quadrilateral Overview

1. Give the most specific name for each shape, and state your reasoning.

   a. 
   
   b. 
   
   c. 

2. Put a check in each box if the shape **ALWAYS** has the given property.

<table>
<thead>
<tr>
<th>Property</th>
<th>Parallelogram</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
<th>Kite</th>
<th>Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sides are congruent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both pairs of opposite sides are congruent</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both pairs of opposite sides are parallel</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both pairs of opposite angles are congruent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consecutive interior angles are supplementary</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exactly one pair of opposite sides are congruent</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>All angles are congruent</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Diagonals are perpendicular</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Diagonals are congruent</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diagonals bisect each other</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Diagonals bisect the interior angles</td>
<td></td>
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</tr>
</tbody>
</table>
3. Create a flow chart that starts with “Quadrilaterals” and shows the classification paths that are possible.

4. Find the measure of each lettered angle.

5. Enter True (T) or False (F) for each of the possible cases in the table.

<table>
<thead>
<tr>
<th>Then it is also a...</th>
<th>If quadrilateral ABCD is a ...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trapezoid</td>
</tr>
<tr>
<td>Trapezoid</td>
<td></td>
</tr>
<tr>
<td>Parallelogram</td>
<td></td>
</tr>
<tr>
<td>Rhombus</td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td></td>
</tr>
</tbody>
</table>

6. If quadrilateral ABCD is a square and BE = BG, find the value of $m\angle DFG$. Provide reasons.

7. If JKLM is a rhombus and KL = LH, find the value of $x$. Provide reasons.
6.4 Quadrilateral Proofs

1. Given: Parallelogram WERT
    Prove: WR and ET bisect each other

Proof:

\begin{align*}
\text{Statement} & \quad \text{Reason} \\
1. & \text{WERT is a parallelogram} & 1. \\
2. & \text{WE} \parallel \text{RT} & 2. \\
3. & \angle 1 \equiv \angle 4, \quad \angle 2 \equiv \angle 3 & 3. \\
4. & \text{WE} \equiv \text{RT} & 4. \\
5. & \quad & 5. \text{ASA} \\
6. & \text{WF} \equiv \text{RF}, \quad \text{EF} \equiv \text{TF} & 6. \\
7. & \quad & 7. \text{Definition of bisector}
\end{align*}

2. Given: \( \angle A \equiv \angle C, \quad \angle B \equiv \angle D \)
    Prove: ABCD is a parallelogram

Proof:

\begin{align*}
\text{Statement} & \quad \text{Reason} \\
1. & \angle A \equiv \angle C, \quad \angle B \equiv \angle D & 1. \\
2. & x^\circ + y^\circ + x^\circ + y^\circ = 360^\circ & 2. \\
3. & 2(x + y)^\circ = 360^\circ & 3. \\
4. & (x + y)^\circ = 180^\circ & 4. \\
5. & \angle A \text{ and } \angle \text{ are supplementary}, \quad \angle B \text{ and } \angle \text{ are supplementary} & 5. \\
6. & \text{AB} \parallel \text{__, BC} \parallel \text{__} & 6. \\
7. & \quad & 7.
\end{align*}
3. Given: Parallelogram ZXCv, Parallelogram BNMC
   Prove: \( \angle Z \cong \angle N \)

4. Given: Parallelogram ABCD
   Prove: \( \triangle AEB \cong \triangle CED \)
   (Assume you do not know any diagonal properties.)

5. Given: Rectangle ABCD
   Prove: \( \triangle ABC \cong \triangle CDA \)
   (Assume you do not know any diagonal properties.)

6. Given: Rectangle ABCD
   Prove: \( \overline{AC} \cong \overline{BD} \)

7. Given: Parallelogram ABCD, \( \overline{BC} \) bisects \( \overline{DP} \)
   Prove: \( \overline{CF} \cong \overline{BF} \)

8. Given: Parallelogram ABCD, \( CP = BC \)
   Prove: \( \angle ADC \cong \angle PCD \)

9. Given: Rhombus ABCD,
   \( F \) is the midpoint of \( \overline{DC} \)
   \( G \) is the midpoint of \( \overline{BC} \)
   Prove: Quadrilateral AGCF is a kite
6.5 Planning Coordinate Proofs

1. ABCD is a kite. Fill in the missing information.

2. A triangle has points at Y(8, 9) and N(2, 9).
   a. Name a point that would make this triangle equilateral.
   b. Name a point that would make this triangle isosceles but not equilateral.

3. Fill in the missing coordinates for each given type of quadrilateral.
   a. ABDC is a square.
   b. ABCD is a rhombus.
   c. ABDC is a parallelogram.
   d. ABCD is an isosceles trapezoid.
4. Fill in the missing coordinates in terms of the variables provided in the diagram.
   a. ABDC is a parallelogram.
   b. ABD is an isosceles triangle.
   c. ABCD is a rectangle.
   d. ABDC is an isosceles trapezoid.
6.6 Coordinate Proofs

Write a coordinate proof for each of the problems. Make a coordinate drawing for each first.

1. Prove that the medians to the legs of an isosceles triangle are congruent. Use the diagram below with the given coordinates.

![Diagram of an isosceles triangle with medians drawn]

2. Given: ABCO is a rectangle with the labeled information shown at right.
   a. Find the coordinates of A, B, C, and O.
   b. Find the coordinates of M, N, P, and Q, which are the midpoints of the sides of the rectangle.
   c. Find the slopes of MN, QP, MQ, and NP.
   d. Based on the slopes, what can be concluded about quadrilateral MNPQ?
   e. Find the lengths of MN, QP, MQ, and NP.
   f. Based on the side lengths, what can be concluded about quadrilateral MNPQ?
3. Prove that the midsegment of a trapezoid is parallel to its bases. Use the diagram below with the given coordinates.

4. Prove that the length of a midsegment of a trapezoid is the average of its two bases. Use the diagram from Problem 4.

5. Prove that the diagonals of a square are perpendicular. Use the diagram at right with the given coordinates.

6. Prove that the diagonals of an isosceles trapezoid are congruent. Use the diagram below with the given coordinates.

7. Given: Right \( \Delta ABC \) with hypotenuse \( \overline{AB} \) shown at right.
   a. Find the coordinates of the midpoint of the hypotenuse.
   b. Prove that the midpoint of the hypotenuse is equidistant from the other three vertices.
8. Prove that if a point lies on the perpendicular bisector of a segment, then the point is equidistant from both endpoints of the segment. (First make your own coordinate drawing.)

9. Show that points (7, 11), (7, −13), and (14, 4) lie on a circle with its center at (2, −1).

10. Two of the vertices of an equilateral triangle are (2, 1) and (6, 5). Find the possible coordinates of the remaining vertex.

11. Prove that in any convex quadrilateral the sum of the squares of the sides is equal to the sum of the squares of the diagonals plus four times the square of the segment joining the midpoints of the diagonals:
   a. Start off by restating what the problem in terms of the diagram below.
   b. Write a proof based on the statement you wrote for part a.
6.7 Introduction to Polygons

1. Copy and complete the table below for regular polygons.

<table>
<thead>
<tr>
<th>Name</th>
<th># of sides</th>
<th>Sum of Interior Angles</th>
<th>Each Interior Angle Measure</th>
<th>Each Exterior Angle Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pentagon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td></td>
<td></td>
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<tr>
<td>Heptagon</td>
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</tr>
<tr>
<td>Octagon</td>
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<td></td>
</tr>
<tr>
<td>Nonagon</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Decagon</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Dodecagon</td>
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<td>:</td>
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</tr>
<tr>
<td>Icosagon</td>
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<td></td>
</tr>
<tr>
<td>n-gon</td>
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<td></td>
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</tr>
</tbody>
</table>

2. Each interior angle of a certain regular polygon has measure 176.4°. How many sides does the polygon have?

3. Each exterior angle of a regular polygon has a measure of 2°. How many sides does the polygon have?

4. The sum of the interior angles of a polygon is three times the sum of the exterior angles. How many sides does the polygon have?

5. Points U, V, W, X, Y, and Z are the vertices of a regular hexagon and also trisect the sides of the large equilateral triangles shown. Given that the area of UVWXYZ is 24, what is the total area of the shaded regions?
6. **Construct** a regular octagon.

7. A regular pentagon and a regular octagon share a common side $\overline{AB}$, as shown. What is the degree measure of $\angle IAE$?

8. The measures of the angles of an octagon are in the ratios of $3:3:3:4:4:4:5:5$. What is the number of degrees in the measure of its largest angle?

9. Explain why it is not possible for a regular polygon to have an interior angle measuring $136^\circ$.

10. Let $ABCDE$ be a regular pentagon and let $ABIHGF$ be a regular hexagon as shown in the diagram. Find the measure of $\angle CBI$. 
6.8 GeoGebra: Star Polygons

If you were to arrange a set of points around a circle and connect each point to its nearest neighbor, you should get a nice convex polygon. (If the points were evenly spaced around a circle, you should get a nice regular convex polygon.) What would happen if you connect every second point, or every third point? This has been done for 5 points on a circle below.
Of course we cannot connect every 5\(^{th}\) point because there are only 5 points.

1. The first and last figures shown the same pentagon. Why is this happening?
2. The second and third figures show the same star polygon. Why is this happening?
3. Conjecture about the sum of the measures of the angles in the star points?

Let’s do some exploration using GeoGebra.

4. Create a circle that fills the work area.
5. Create 5 points that lie on the circle. The points do not have to be equally spaced.
6. Connect each point to its nearest neighbor. This should make a pentagon. Measure each angle of the pentagon. Sum the five angle measures that you found. Why is the sum not surprising to you?
7. Delete the segments you made in step 6. Now connect every 2\(^{nd}\) point. Measure each angle of this star polygon. Sum the five angle measures that you found.
8. Repeat part 6 for every 3\(^{rd}\) point and every 4\(^{th}\) point.
9. Is there any pattern that you can notice in your star polygon?
10. Let’s repeat the exploration using a hexagon, a heptagon, an octagon, a nonagon, a decagon, an undecagon, and a dodecagon. Fill in the information in the table below.

<table>
<thead>
<tr>
<th>Number of Star Points</th>
<th>Every 2(^{nd}) Point</th>
<th>Every 3(^{rd}) Point</th>
<th>Every 4(^{th}) Point</th>
<th>Every 5(^{th}) Point</th>
<th>Every 6(^{th}) Point</th>
<th>Every 7(^{th}) Point</th>
<th>Every 8(^{th}) Point</th>
<th>Every 9(^{th}) Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>540°</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>6</td>
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<td>7</td>
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</table>

11. Use your table to come up with a rule to find the sum of the measures of the angles in \(n\)-point stars.

12. Write a short report describing the conjectures you made and the patterns you found in the chart.
6.9 Polygon Problems

1. Call the figure to the right a **regular semioctagon**.
   a. What do you think this name means?
   b. Find the values of $x$ and $y$.

2. If the measure of an exterior angle of a regular polygon is $15^\circ$, how many sides does the polygon have?

3. Find the number of diagonals that can be drawn in a **pentadecagon**.

4. Tell whether each statement is Sometimes True, Always True, or Never True. Clearly explain your answers.
   a. An equiangular triangle is isosceles.
   b. The number of diagonals in a polygon is the same as the number of sides.
   c. An exterior angle of a triangle is larger in measure than any angle of a triangle.
   d. One of the base angles of an isosceles triangle has a measure greater than that of one of the exterior angles of the triangle.

5. An arithmetic sequence is a sequence of terms in which the difference between any two consecutive terms is always the same. (For example, 1, 5, 9, 13, 17 form an arithmetic sequence with the difference of 4.) Do the numbers of diagonals in a triangle, a quadrilateral, a pentagon, and a hexagon form an arithmetic sequence? Be sure to fully support your answer.

6. In the figure shown at right, show that $h = \frac{1}{2}(b + d)$.
7. Find the area of a pentagon ABCDE given the side lengths and right angles shown.

8. In the star diagram, find the sum $m\angle A + m\angle C + m\angle E + m\angle G + m\angle I$ by using a protractor to measure the angles.

9. Given: ABCDEF is a regular hexagon. Prove: ACDF is a rectangle.

10. Given: PENTA is a regular pentagon. Prove: ΔPNT is isosceles.